8

MEASUREMENT AND GEOMETRY

CONGRUENT FIGURES

A **tessellation** is a 'tiling pattern' of repeated congruent shapes covering a flat surface so that there is no overlapping or gaps. Regular tessellations use only one shape and use the 3 regular polygons, an equilateral triangle, the square and the regular hexagon.

M. C. Escher (1898 – 1972) was a Dutch graphic artist who used congruent figures to create many interesting tessellations using birds, fish and other creatures. These are called non-regular tessellations.

Chapter outline

	Working mathematically				
8.01 Transformations	U	F		R	C
8.02 Congruent figures	U	F		R	C
8.03 Constructing triangles	U	F	PS	R	C
8.04 Tests for congruent triangles	U	F	PS	R	C
8.05 Proving properties of quadrilaterals	U	F	PS	R	C
8.06 Extension: Bisecting intervals and angles *	U	F		R	C

***Extension**

Wordbank

congruence test One of 4 tests for proving that triangles are congruent: SSS, SAS, AAS and RHS

congruent Identical; exactly the same (symbol: ≡)

image A transformed shape after it has been translated, reflected or rotated

included angle The angle between 2 given sides of a shape

RHS The congruence test 'right angle-hypotenuse-side'

SAS The congruence test 'side-angle-side', where the included angle is between the 2 sides

superimpose To place one figure on top of another congruent figure so that sides and angles match

transformations Translations (slides), reflections (flips) or rotations (spins or turns)

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- translate, reflect and rotate shapes, including on a number plane using coordinates, where the reflections are across the x- or y-axis and the rotations are of multiples of 90°
- define congruent figures using transformations
- use the symbol '≡' in a congruence statement, naming the vertices in matching order
- investigate minimum conditions for constructing triangles
- identify the congruent triangles tests (SSS, SAS, AAS, RHS)
- investigate and prove the properties of the special quadrilaterals using congruent triangles
- (EXTENSION) bisect angles with a pair of compasses and a ruler, using properties of congruent triangles

SkillCheck ANSWERS ON P. 569

1 Draw each special quadrilateral and mark all axes of symmetry.

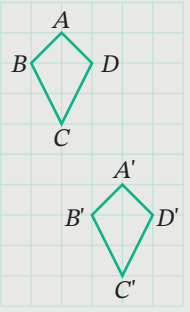
a rectangle **b** square **c** parallelogram
d rhombus **e** trapezium **f** kite

2 From the list in Question **1**, name **all** quadrilaterals that have:

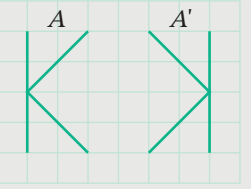
a all angles 90° **b** opposite sides parallel
c opposite sides equal **d** one pair of parallel sides
e 4 equal sides **f** 2 pairs of adjacent sides equal

3 State the transformation (translation, rotation or reflection) used on each original figure to form the image.

a



b



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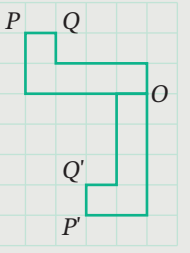
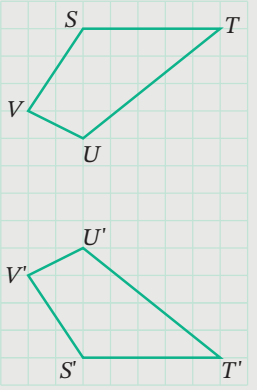
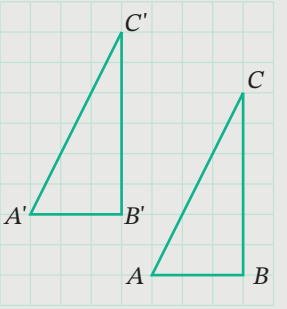


d



e

f



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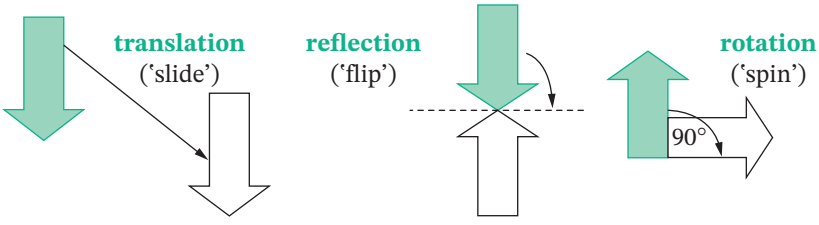
Transformations

8.01

There are 3 congruence **transformations**.



'Congruence' means identical and 'transformation' means change. Even though a shape has changed position (transformed), it still has the same shape and size (congruence).

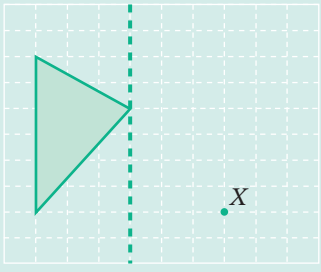
- The original shape is called the **original**
- The transformed shape is called the **image**

Composite transformations

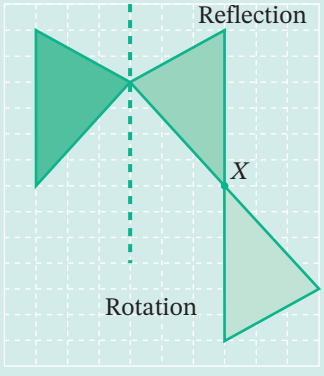
Combinations of translations, reflections and rotations can be applied one after the other. These are called **composite transformations**.

Example 1

Reflect the triangle below across the dotted line and then rotate the image 180° about X.



Solution



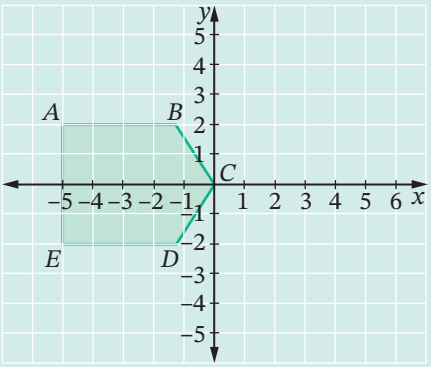
Transformations on a number plane

When a point or vertex P of an original shape is transformed, the corresponding point or vertex on the image is labelled P', pronounced 'P-dash' or 'P-prime'.

Example 2

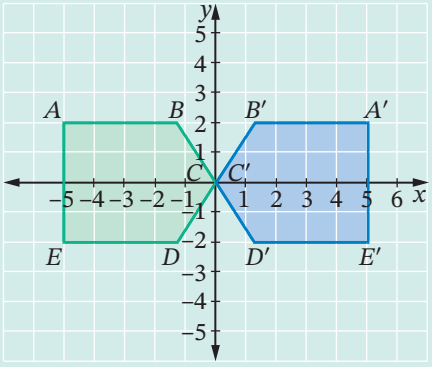
a Reflect the pentagon $ABCDE$ across the y-axis to create the image $A'B'C'D'E'$.

b Compare the coordinates of each vertex of $ABCDE$ to its matching vertex in $A'B'C'D'E'$.



Solution

a



b $A(-5, 2) \rightarrow A'(5, 2)$

$B(-1, 2) \rightarrow B'(1, 2)$

$C(0, 0) \rightarrow C'(0, 0)$

$D(-1, -2) \rightarrow D'(1, -2)$

$E(-5, -2) \rightarrow E'(5, -2)$

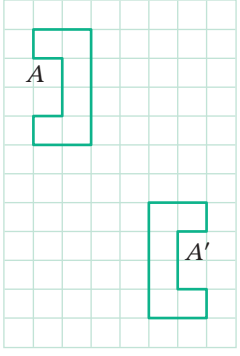
When reflected in the y-axis, the x-coordinate of each vertex changes sign while the y-coordinate stays the same.

EXERCISE 8.01 ANSWERS ON P. 569

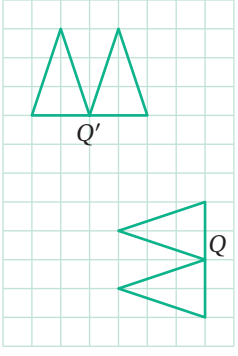
Transformations UFRC

1 For each diagram, state the combination of 2 transformations used on the original figure to form the image. R

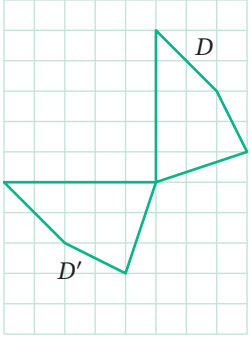
a



b



c



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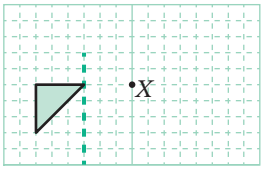
EXAMPLE 1

WS 5 mm grid paper

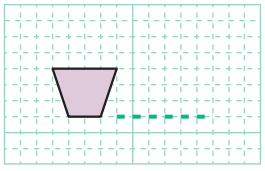
8.01

2 Copy each shape onto grid paper and draw the image after performing the stated composite transformations.

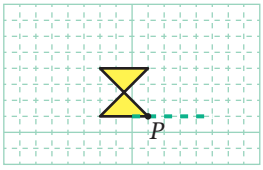
a Reflect across the line, rotate 90° clockwise about X.



b Translate 5 units right, then reflect across the line.



c Rotate 90° clockwise about P, then reflect across the line, then translate 3 units left.

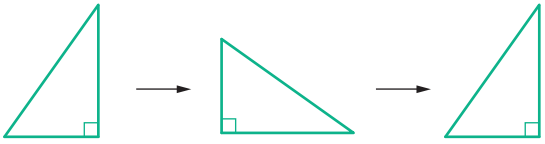


3 Each diagram below has been transformed twice. R

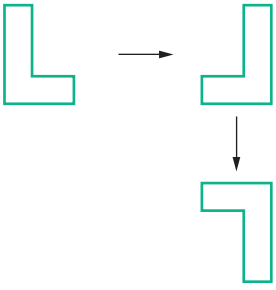
i Name the 2 transformations that have been performed.

ii Name one transformation that would give the same result as the 2 transformations.

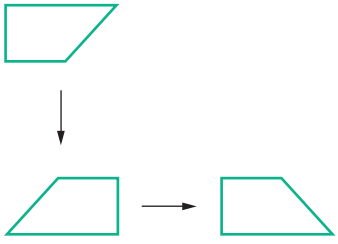
a



b

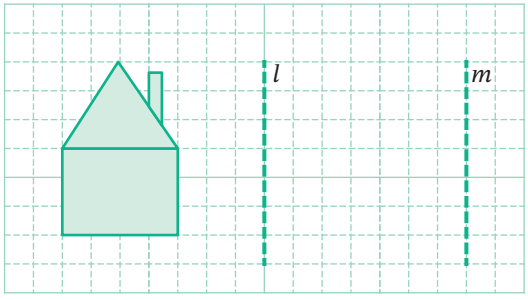


c



4 **a** Reflect the house shape across line l and then across line m.

b Which single transformation would give the same result as the 2 reflections in part **a**? R



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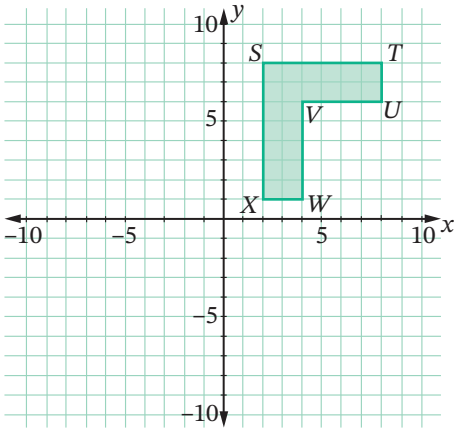
EXAMPLE 2

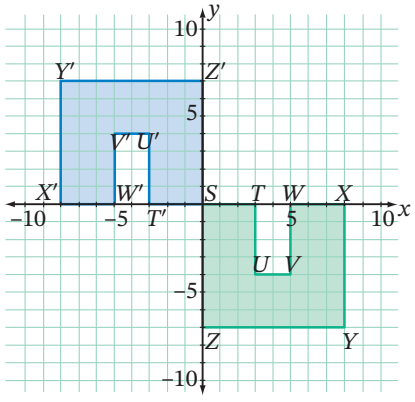
5 **a** Copy and reflect the L-shape across the x-axis to create the image $S'T'U'V'W'X'$.

b Compare the coordinates of each vertex in the original figure to its matching vertex in the image figure. R C



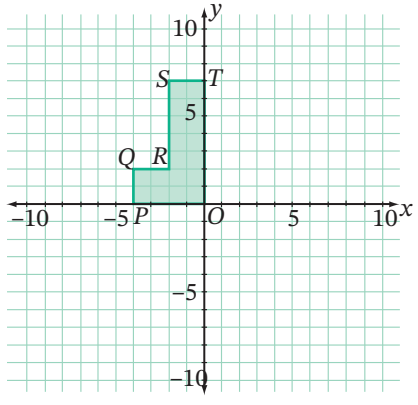
6 **a** Accurately describe the transformation that has been performed on $STUVWXYZ$.

b Compare the coordinates of each vertex in the original shape to its matching vertex in the image. R C



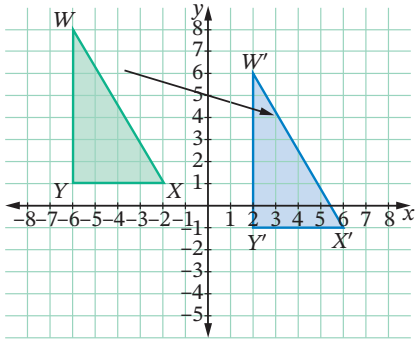
7 **a** Copy and rotate $OPQRST$ about O through an angle of 180°.

b Write the coordinates of the new position of Q and compare them to its original coordinates. R C



8 **a** Accurately describe the transformation that has been performed on $\triangle WXY$.

b Compare the coordinates of the vertices of $\triangle WXY$ to those of the image $\triangle W'X'Y'$. R C



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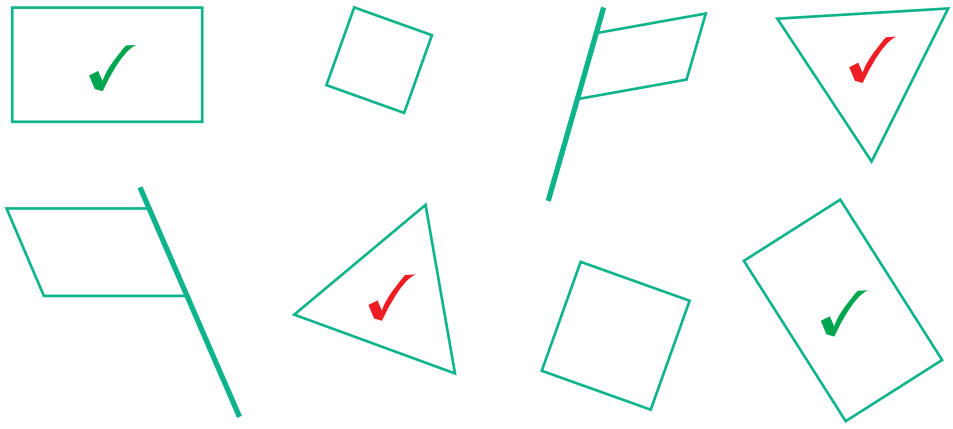
Congruent figures

8.02

Two shapes are **congruent** if they have exactly the same shape and size.
Congruent means 'identical'.

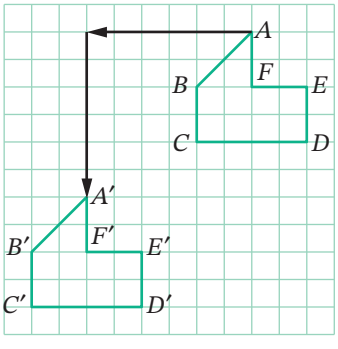


- The 2 rectangles are congruent (marked by green ticks)
- The 2 triangles are congruent (marked by red ticks)
- The 2 squares are not congruent, because they are not the same size
- The 2 flags are not congruent, because they are not the same size

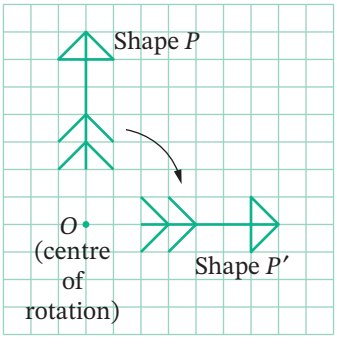
One way of testing whether 2 shapes are congruent is to **superimpose** one shape onto the other, that is, to move it to a position on top of the other so that the sides and angles match.

Congruent figures may be identified by superimposing them through a combination of translations, rotations and reflections.

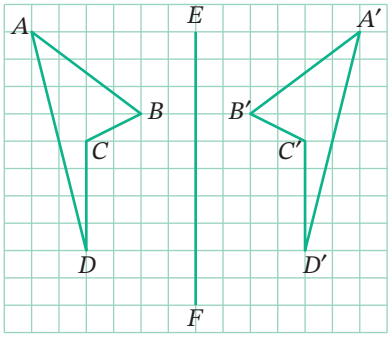
These 2 figures are congruent because *ABCDEF* can be superimposed onto *A′B′C′D′E′F′* by translation.



These 2 figures are congruent because arrow *P* can be superimposed onto arrow *P′* by rotation.

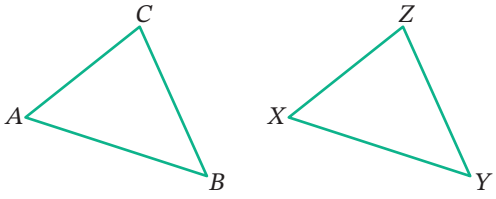


These 2 quadrilaterals are congruent because $ABCD$ can be superimposed onto $A'B'C'D'$ by reflection.



The congruence symbol ≡

The symbol for 'is congruent to' is a special equals sign, written as '≡' (which also means 'is identical to'). The 2 triangles at right are congruent, so we can write $\triangle ABC \equiv \triangle XYZ$, which is read 'triangle ABC is congruent to triangle XYZ'.

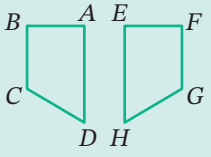


When using this notation, we must make sure that the vertices (angles) of the congruent figures are written in matching order: $\angle A = \angle X$, $\angle B = \angle Y$, $\angle C = \angle Z$.

Example 3

For each pair of congruent figures, identify the required matching side and angle, then write the statement relating the figures using the congruency symbol.

a



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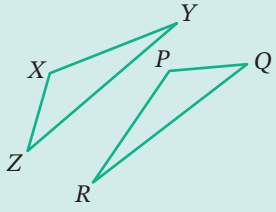

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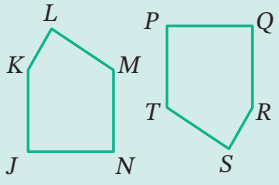


Solution

a $\angle B = \angle F$, $CD = GH$, $ABCD \equiv EFGH$

b $PQ = XZ$, $\angle R = \angle Y$, $\triangle XYZ \equiv \triangle PRQ$

c $\angle J = \angle Q$, $TS = ML$, $JKLMN \equiv QRSTP$

Note that the vertices of congruent figures are named in matching order: equal angles are paired.

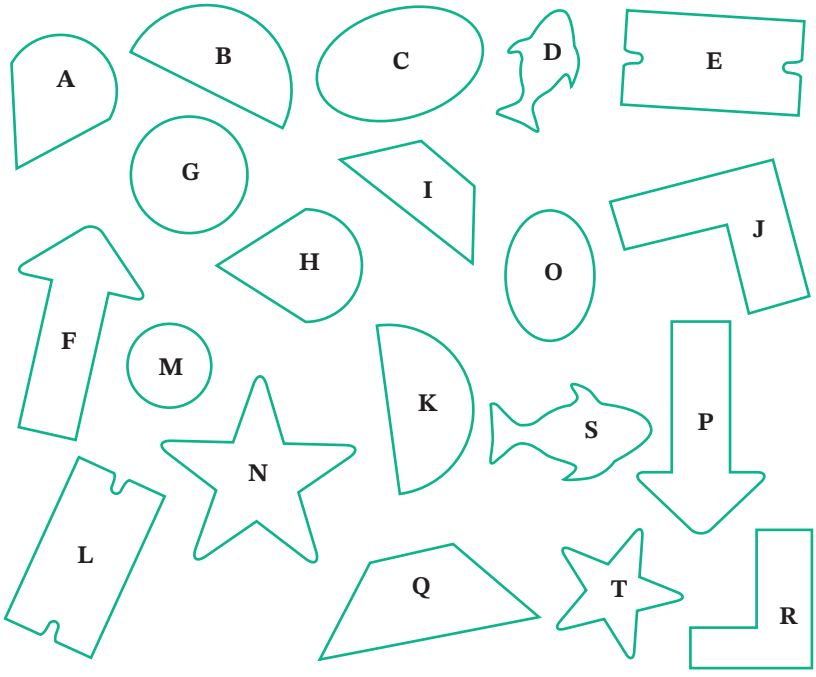
Congruent figures

- Matching sides are equal
- Matching angles are equal
- The symbol for 'is congruent to' is ≡
- Vertices are named in matching order.

EXERCISE 8.02 ANSWERS ON P. 570

Congruent figures UFRC

1 By measuring or examining, find the 4 matching pairs of congruent shapes from the diagrams below.

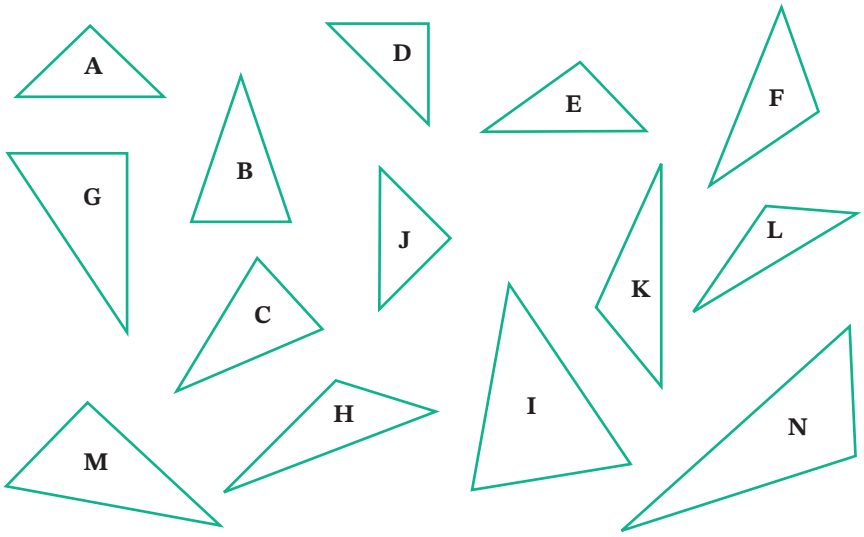


2 Of the triangles below:

a which 2 triangles are congruent to triangle A?

b which triangle is congruent to triangle B?

c which triangle is congruent to triangle K?



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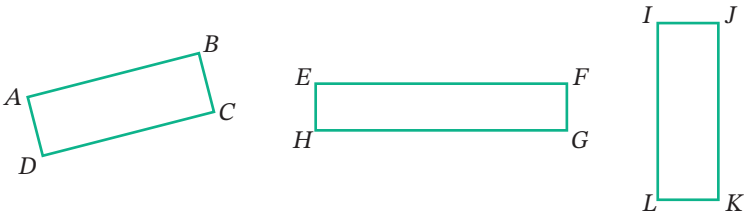
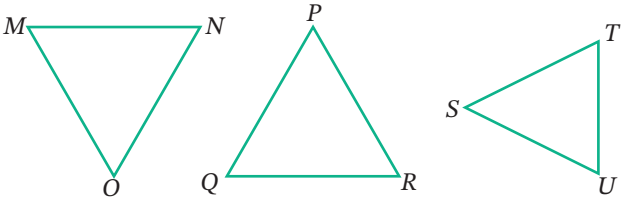
EXAMPLE 3

3 For each set of shapes, identify the 2 congruent figures, using the correct notation and matching order of vertices. R C

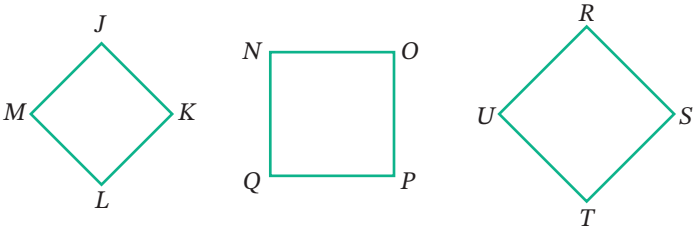
a



b



c



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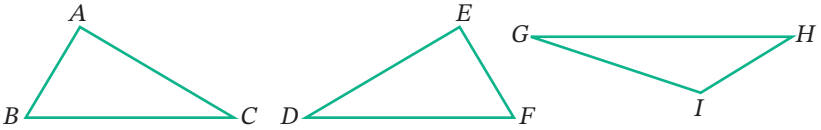

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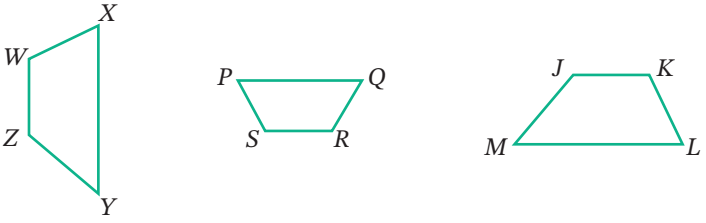

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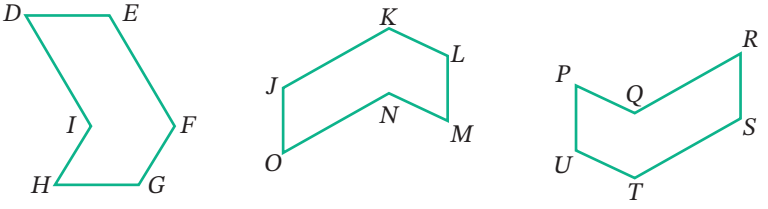
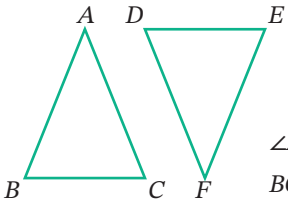


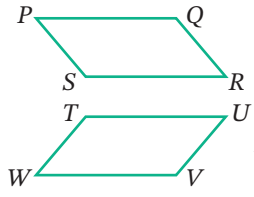
4 Copy and complete the matching angle and side in each pair of congruent figures. R C

a



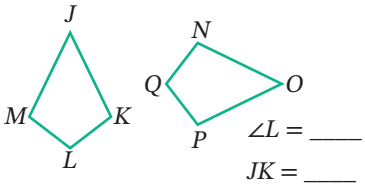
$\angle A =$ ____
$BC =$ ____

b



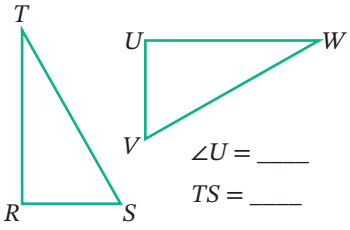
$\angle W =$ ____
$UV =$ ____

c



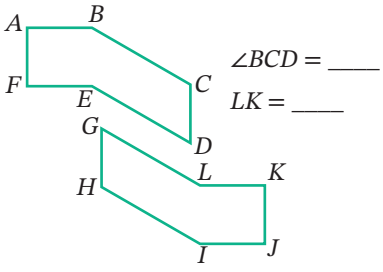
$\angle L =$ ____
$JK =$ ____

d



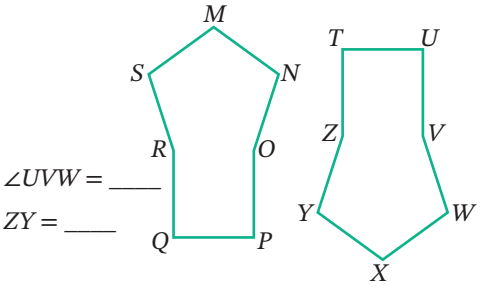
$\angle U =$ ____
$TS =$ ____

e



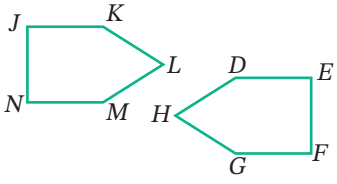
$\angle BCD =$ ____
$LK =$ ____

f



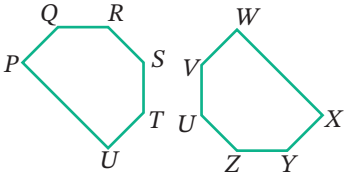
$\angle UVW =$ ____
$ZY =$ ____

g



$\angle NML =$ ____
$EF =$ ____

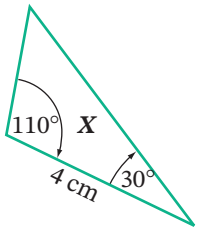
h

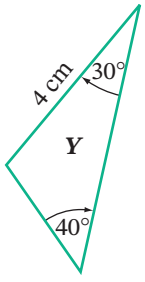


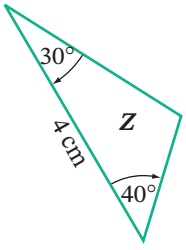
$\angle QPU =$ ____
$RS =$ ____

5 Which triangles below are congruent? Select the correct answer **A**, **B**, **C** or **D**. R

A *X* and *Y* **B** *X* and *Z* **C** *Y* and *Z* **D** *X*, *Y* and *Z*







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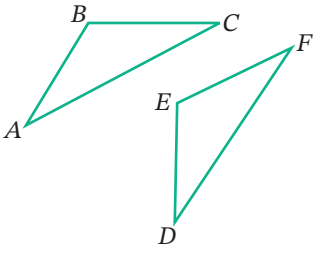
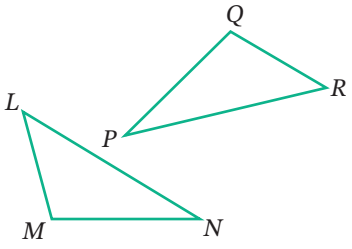
6 For each pair of congruent triangles, list: R C

i the 3 pairs of matching sides **ii** the 3 pairs of matching angles.

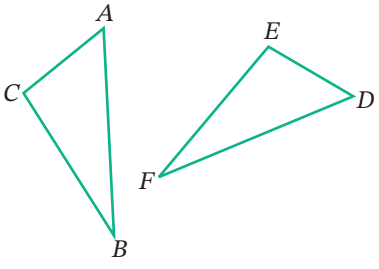
a



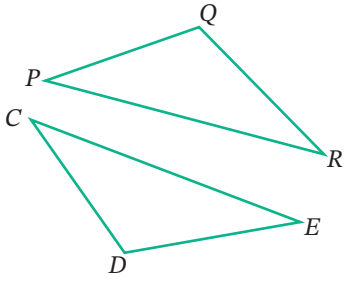
b



c



d



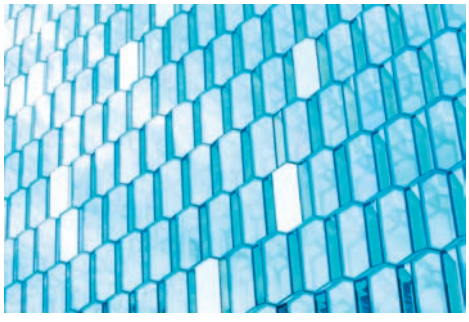
7 Draw a rectangle and its diagonals. You now have 4 triangles. Shade congruent triangles using the same colours.

8 Draw a parallelogram and its diagonals. You now have 4 triangles. Shade congruent triangles using the same colours.

Investigation

Congruence in design

Congruent figures appear in a variety of art and design work. Some examples are shown below.



iStock.com/powerofforever

iStock.com/AlizadaStudios

1 Find 5 examples of congruent figures in the real world. Look at logos and design; artwork, including the work of Escher; tessellations in paving and walls; and decoration in other cultures, including Aboriginal and Islamic design. Put together a presentation of your examples, showing clearly the congruent shapes that have been used.

2 Produce your own design or piece of art based on congruent shapes.

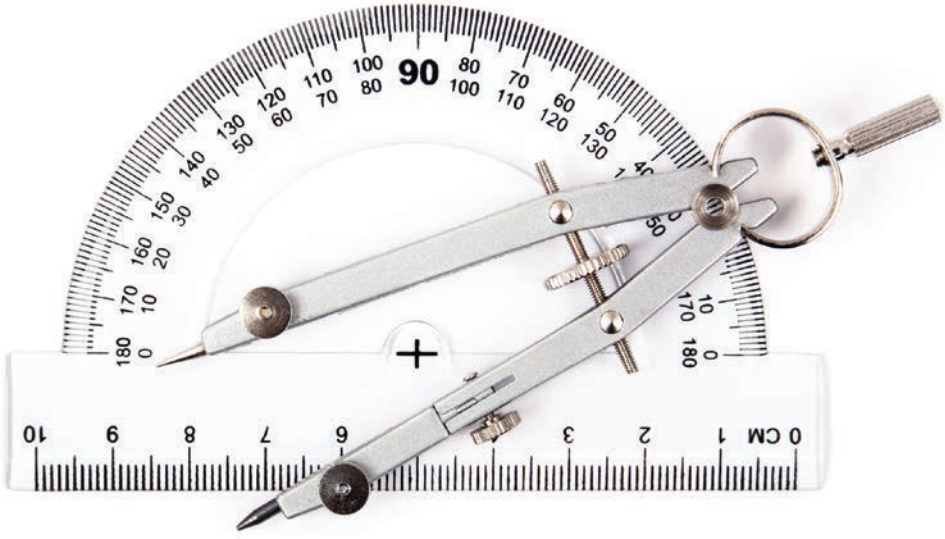
Constructing triangles

8.03

To construct (draw accurately) a triangle, we need to know the length of its sides and the size of its angles. We can construct triangles using geometrical instruments (ruler, protractor and compasses) or dynamic geometry software. *Hint*: Draw a rough sketch before beginning the construction.

WS Drawing different triangles

Triangle constructions group clues



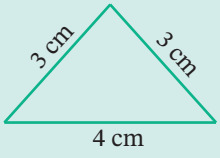
iStock.com/ConstantinosZ

Example 4

Construct an isosceles triangle with sides of length 3 cm, 3 cm and 4 cm.

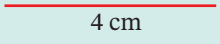
Solution

Step 1:



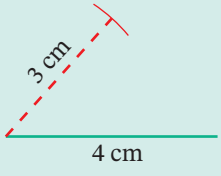
Do a rough sketch.

Step 2:



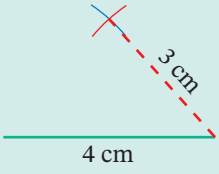
Use a ruler.

Step 3:



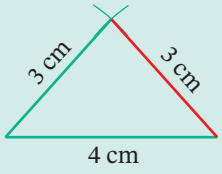
Use a ruler and compasses.

Step 4:



Use a ruler and compasses.

Step 5:



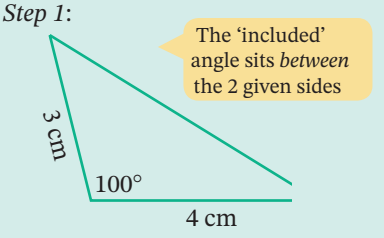
Use a ruler to complete the triangle.

Example 5

Construct a triangle with 2 sides of length 3 cm and 4 cm and an **included angle** of 100°.

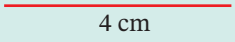
Solution

Step 1:



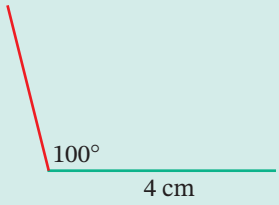
Do a rough sketch.

Step 2:



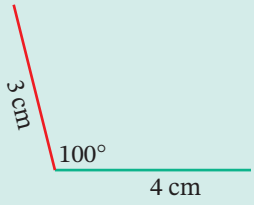
Use a ruler to draw the 4 cm side.

Step 3:



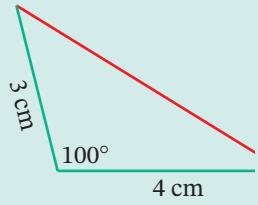
Use a protractor to measure 100°.

Step 4:



Use a ruler to draw the 3 cm side.

Step 5:



Complete the triangle.

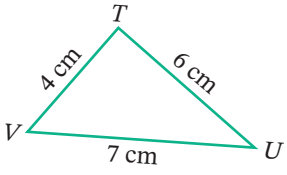
EXERCISE 8.03 ANSWERS ON P. 570

Constructing triangles U F PS R C

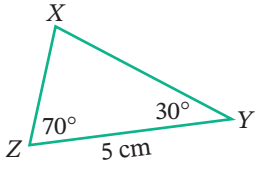
EXAMPLE 4, 5

1 Construct each triangle accurately.

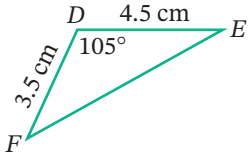
a



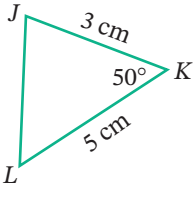
b



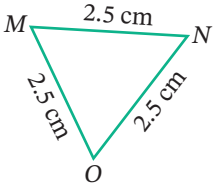
c



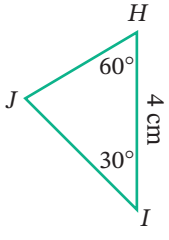
d



e



f



2 **a** Which triangle in question **1** is equilateral?

b What is the size of each angle?

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9780170453141

3 **a** Which triangle in question **1** is right-angled?

b Measure the length of its shortest side.

4 How many measurements about the sides and angles of a triangle are needed to construct a congruent triangle? As a group activity for 3–4 students, for each triangle described below, try to construct different triangles (if possible) with the same measurements. If you can't, and all the triangles drawn are **congruent**, then the measurements that were given are enough. But if one or more of the triangles drawn are different, then the measurements that were given are not enough.

A similar investigation using dynamic geometry software can be found in the Technology activity on page 354.

8.03

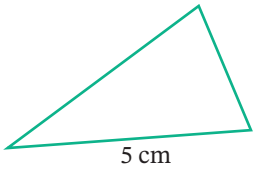
For each triangle described: PS R

i each group member constructs the triangle with the given measurements

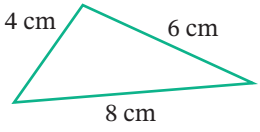
ii determine whether your triangle is 'congruent to' or 'different from' the triangles made by the others in your group (you may need to cut them out first to superimpose them)

iii label the shape 'congruent' or 'different' and paste it into your book.

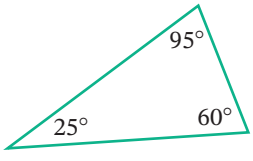
a one side and one angle: 5 cm and any angle 35°



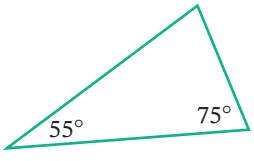
b 3 sides: 4 cm, 6 cm and 8 cm



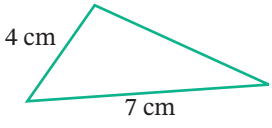
c 3 angles: 25°, 95° and 60°



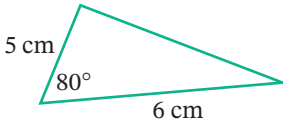
d 2 angles and one side: 55°, 75° and any side 8 cm



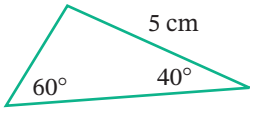
e 2 sides and one angle: 4 cm, 7 cm and any angle 40°



f 2 sides and included angle: 5 cm, 6 cm and included angle 80°



g 2 angles and particular side: 60°, 40° and the side opposite the 60° angle 5 cm



5 Consider the triangles you constructed in question **4**. Which set of measurements produced congruent triangles? Suggest a reason why. R C

Mental skills 8A: Maths without calculators ANSWERS ON P. 570

Time before and time after

1 Study each example.

a What is the time 7 hours and 40 minutes after 11:45 a.m.?

11:45 a.m. + 7 hours = 6:45 p.m.

Count: '11:45, 12:45, 1:45, 2:45, 3:45, 4:45, 5:45, 6:45'

6:45 p.m. + 40 minutes = 6:45 p.m. + 15 minutes + 25 minutes
= 7:00 p.m. + 25 minutes
= 7:25 p.m.

OR:

b What is the time 10 hours and 15 minutes after 18:50?

18:50 + 10 hours = 04:50 (next day).

Count: '18:50, 19:50, 20:50, 21:50, 22:50, 23:50, 00:50, 01:50, 02:50, 03:50, 04:50'

04:50 + 15 min = 04:50 hours + 10 min + 5 min
= 05:00 + 5 min
= 05:05 (next day)

OR:

2 Now find the time of day:

a 3 hours 20 minutes after 9:05 a.m.

b 5 hours 40 minutes after 7:30 p.m.

c 4 hours 35 minutes after 6:15 p.m.

d 11 hours 10 minutes after 11:45 a.m.

e 2 hours 45 minutes after 03:25

f 7 hours 5 minutes after 17:05

g 8 hours 30 minutes after 12:40 a.m.

h 4 hours 55 minutes after 10:20 p.m.

i 6 hours 25 minutes after 04:35

j 2 hours 15 minutes after 20:50

k 9 hours 50 minutes after 2:30 p.m.

l 3 hours 10 minutes after 8:25 a.m.

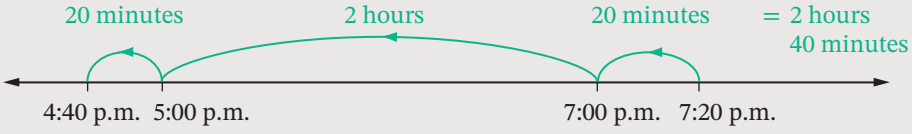
3 Study each example.

a What is the time 2 hours and 40 minutes before 7:20 p.m.?

7:20 p.m. − 2 hours = 5:20 p.m. Count back: '7:20, 6:20, 5:20'

5 : 20 p.m. − 40 minutes = 5 : 20 p.m. − 20 minutes − 20 minutes
= 5 : 00 p.m. − 20 minutes
= 4 : 40 p.m.

OR:



b What is the time 8 hours and 45 minutes before 11:15?

11:15 − 8 hours = 03:15

Count back: '11:15, 10:15, 09:15, 08:15, 07:15, 06:15, 05:15, 04:15, 03:15' (or 11 − 8 = 3).

03 : 15 − 45 min = 03 : 15 − 15 min − 30 min
= 03 : 00 − 30 min
= 02 : 30

OR:

4 Now find the time of day:

a 1 hour 15 minutes before 7:20 p.m.

b 4 hours 40 minutes before 11:20 a.m.

c 3 hours 20 minutes before 3:30 p.m.

d 5 hours 35 minutes before 8:25 a.m.

e 2 hours 10 minutes before 14:55

f 3 hours 45 minutes before 07:40

g 5 hours 25 minutes before 4:15 a.m.

h 9 hours 30 minutes before 9:45 p.m.

i 4 hours 20 minutes before 20:05

j 2 hours 15 minutes before 06:15

Technology

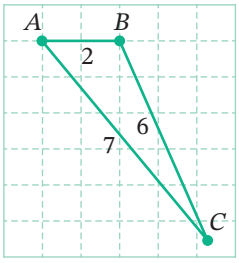
Tests for congruent triangles

We will use dynamic geometry software to test whether 2 triangles are congruent. It is best to do this activity in small groups.

Given 3 sides: SSS

1 Construct this triangle.

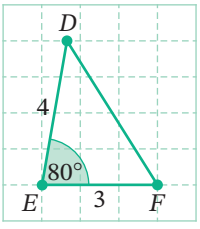
2 Can you construct a **different** triangle with the same 3 side lengths? What do you notice about the second triangle? Is it congruent (exactly the same shape and size) to the first triangle?



Given 2 sides and the included angle: SAS

1 Construct this triangle.

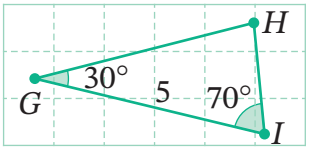
2 Can you construct a different triangle with the same 2 sides and an included angle of 80°? Or is it congruent to the first triangle?



Given 2 angles and a side: AAS

1 Construct this triangle with a side length of 5 units between 2 angles of size 30° and 70°.

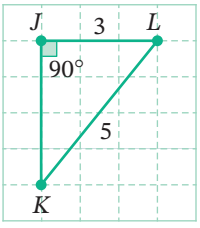
2 Can you construct a different triangle with the same 2 angles and a matching side of 5 cm? Or is it congruent to the first triangle?



Given a right angle, hypotenuse and side: RHS

1 Construct this right-angled triangle with one of the shorter sides being 3 cm and the hypotenuse 5 cm.

2 Can you construct a different triangle with the same 2 sides and right angle? Or is it congruent to the first triangle?

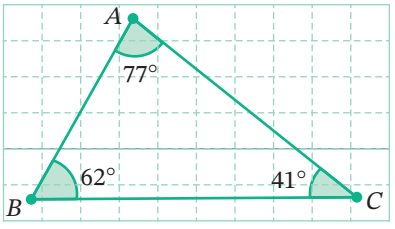


Given 3 angles: AAA

1 Construct this triangle with angles 77°, 62° and 41°.

2 Can you construct a different triangle with the 3 angles? Or is it congruent to the first triangle?

3 Are all triangles with angles 77°, 62° and 41° congruent?



Investigation

Tests for congruent triangles

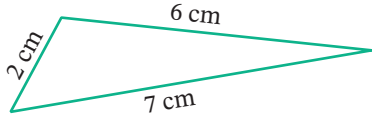
You will need: paper, scissors and geometrical instruments.

How much information about a triangle is necessary for us to be able to draw a congruent triangle? This group investigation will look at the conditions required to show that 2 triangles are congruent, based on your answers to questions from the previous exercise. It is best to complete this investigation in small groups, with every person constructing their own triangle.

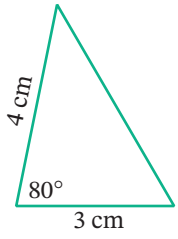
For each triangle described below:

- **a** every person in the group constructs the triangle accurately
- **b** cut the triangle out and compare it with others in the group
- **c** decide whether all of the triangles in your group are congruent.

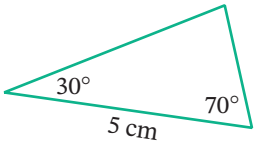
1 Given 3 sides: **SSS**



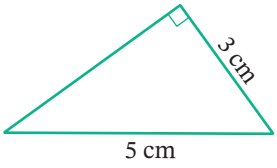
2 Given 2 sides and the included angle: **SAS**



3 Given 2 angles and a side: AAS



4 Given a right angle, hypotenuse and side: RHS



From this investigation, you can see that you don't need to know all the measurements (sides and angles) of 2 triangles to determine whether they are congruent.
Only **3** measurements are needed, which gives us a method of testing for congruent triangles (described in the next section).

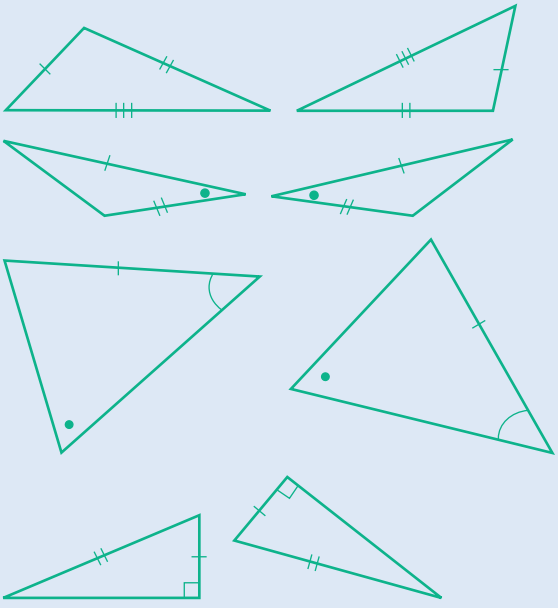
8.04 Tests for congruent triangles

Tests for congruent triangles

There are 4 **congruence tests** for proving congruent triangles: **SSS**, **SAS**, **AAS** or **RHS**.

Two triangles are congruent if:

- the 3 sides of one triangle are equal to the 3 sides of the other triangle **(SSS rule)**
- 2 sides and the **included** angle of one triangle are equal to 2 sides and the **included** angle of the other triangle **(SAS rule)**
- 2 angles and one side of one triangle are respectively equal to 2 angles and the matching side of the other triangle **(AAS rule)**
- they are right-angled and the hypotenuse and another side of one triangle are respectively equal to the hypotenuse and another side of the other triangle **(RHS rule)**.

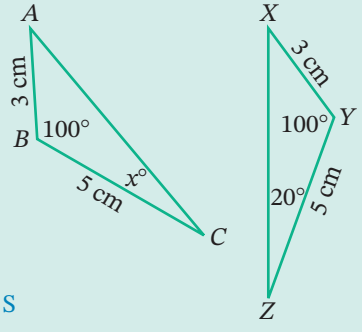


Example 6

a Which test shows that these 2 triangles are congruent?

b Use the correct notation to write a congruency statement relating these 2 triangles.

c Find x.



Solution

a $AB = XY = 3$ cm S

$\angle B = \angle Y = 100°$ A

$BC = YZ = 5$ cm S

The congruence test is SAS.

b $\triangle ABC \equiv \triangle XYZ$ Matching order of vertices.

c $\angle C$ matches $\angle Z$.

$\therefore x = 20$

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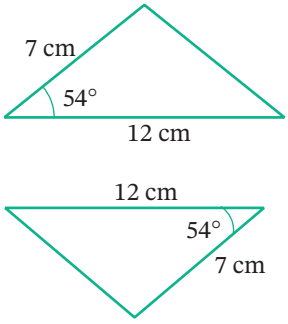
EXERCISE 8.04 ANSWERS ON P. 570

Tests for congruent triangles U F PS R C

EXAMPLE 6

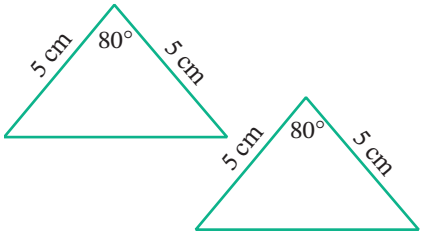
1 Which test can be used to show that these 2 triangles are congruent? Select the correct answer **A**, **B**, **C** or **D**. R C

A SSS **B** SAS
C AAS **D** RHS

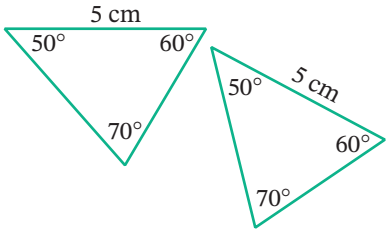


2 For each pair of triangles, state which rule (SSS, SAS, AAS or RHS) shows that they are congruent. R C

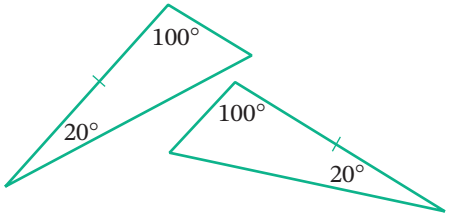
a



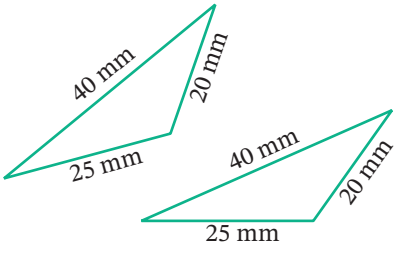
b



c



d



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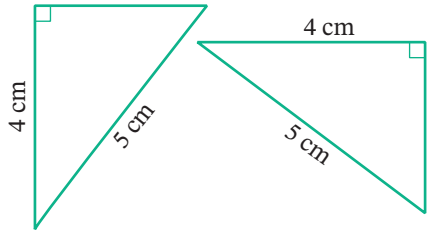
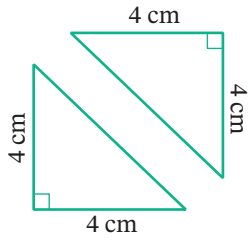
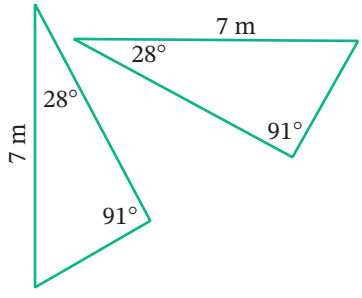


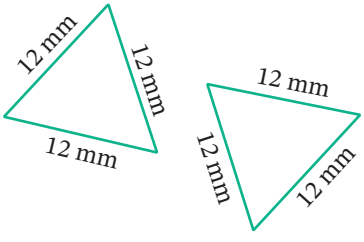
f



g



h



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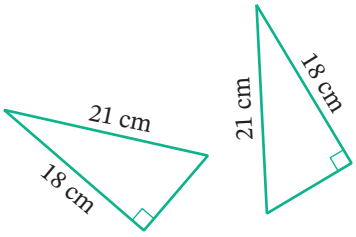
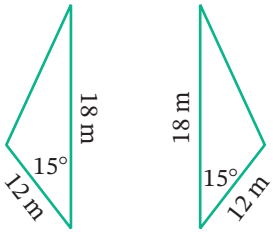
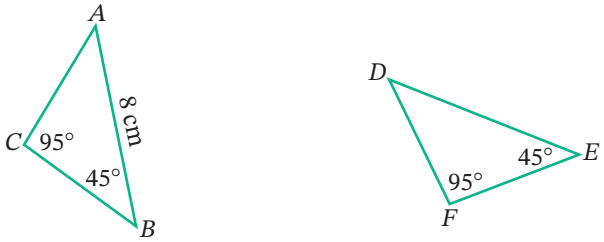
i



j



3 **a** What additional information is needed to prove that these 2 triangles are congruent? Select **A**, **B**, **C** or **D**. R C



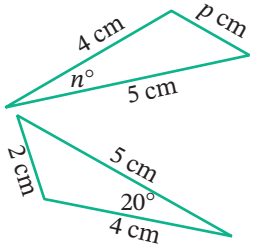
A $\angle EDF = 40°$ **B** $DE = 8$ cm

C $DF = 8$cm **D** $EF = 8$ cm

b Use the correct notation to write a congruency statement relating these 2 triangles. C

4 Find the value of each variable for each pair of congruent triangles. R

a



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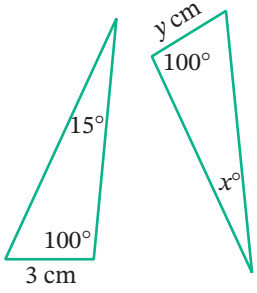
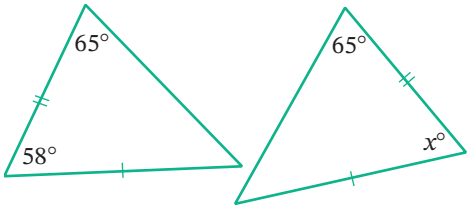
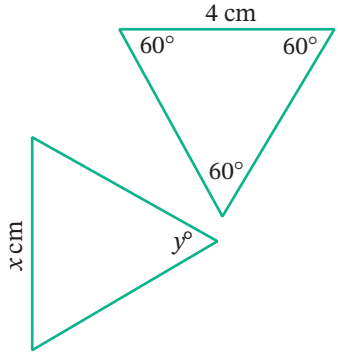


c

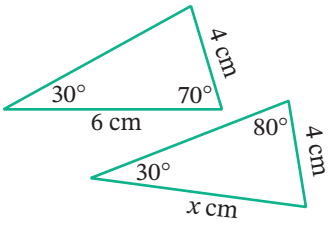


d

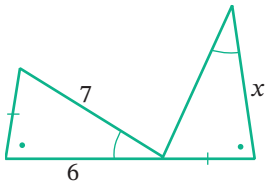


5 For each pair of triangles, state which test can be used to prove that they are congruent and then find the value of the variable. R C

a



b



6 **a** What geometrical rule shows that $x = 46$ in the diagram?

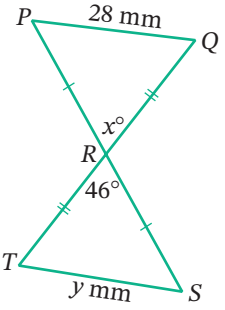
b Which test can be used to prove that the 2 triangles are congruent?

c Use the correct notation to write a congruency statement relating these 2 triangles.

d Find y.

e Which angle in $\triangle RST$ is equal in size to $\angle P$?

f Hence why are sides PQ and TS parallel? PS R C



Did you know?

Quilting

Quilting is the process of stitching together 2 or more layers of fabric, usually a top layer with a design, then the padding for warmth and finally a backing material. Quilting is used to make bedcoverings, wall hangings, clothing and other items.

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iStock.com/MaxCab

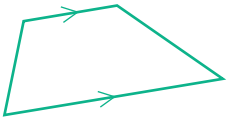
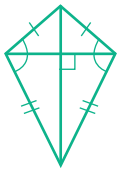
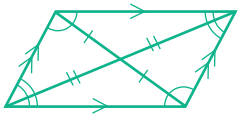
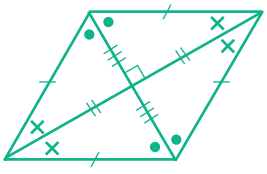
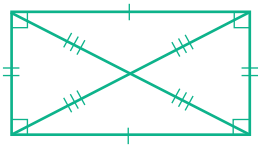
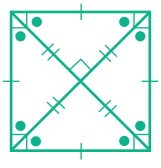
Often blocks of design are sewn together into rows and columns to create a quilt. The design blocks mostly use congruent shapes to create a pattern for the quilt.

Find examples of quilting patterns and highlight the congruent figures in them.

8.05 Proving properties of quadrilaterals

WS
Proving properties of quadrilaterals

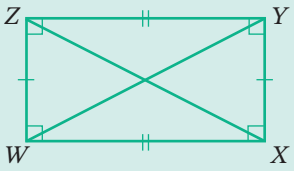
In Chapter 4, *Geometry*, we examined many properties of the special quadrilaterals.

Quadrilateral	Properties
Trapezium 	• One pair of parallel sides • No axes of symmetry
Kite 	• 2 pairs of equal adjacent sides • One pair of opposite angles equal • One axis of symmetry • Diagonals intersect at right angles
Parallelogram 	• Opposite sides are parallel and equal • Opposite angles are equal • No axes of symmetry • Diagonals bisect each other
Rhombus 	• 4 equal sides • 2 axes of symmetry • A special type of parallelogram • Diagonals bisect each other at right angles • Diagonals bisect the angles of the rhombus
Rectangle 	• All 4 angles measure 90° • 2 axes of symmetry • A special type of parallelogram • Diagonals are equal and bisect each other
Square 	• 4 equal sides, 4 angles of 90° • 4 axes of symmetry • A special type of rhombus and rectangle • Diagonals are equal and bisect each other at right angles • Diagonals bisect the angles of the square

We can now use congruent triangles to prove some of these properties.

Example 7

ZYXW is a rectangle, so its opposite sides are equal and all angles are 90°.



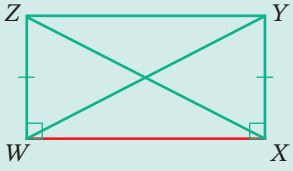
a $\triangle ZXW$ and $\triangle YWX$ are both right-angled triangles that have *WX* as their base. Which test proves that they are congruent?

b Which side is equal to *ZX*?

c What does this prove about the diagonals of a rectangle?

Solution

a $ZW = YX$, $\angle ZWX = \angle YXW = 90°$, *WX* is shared (the common side in red). Therefore, SAS proves that they are congruent.



b *YW* is equal to *ZX*.

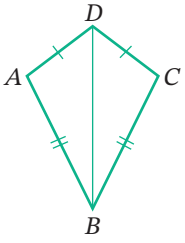
c The diagonals of a rectangle are equal in length.

EXERCISE 8.05 ANSWERS ON P. 571

Proving properties of quadrilaterals U F PS R C

EXAMPLE 7

1 *ABCD* is a kite, so it has 2 pairs of equal adjacent sides. The diagonal *DB* is its axis of symmetry that divides it into 2 congruent triangles. PS R C



a Which test proves that the 2 triangles are congruent?

b Which angle is equal to $\angle A$?

c Which angle is equal to $\angle ADB$? Copy the diagram and mark both angles with a dot.

d Draw the other diagonal *AC*, intersecting *DB* at point *X*.

e This creates 4 triangles. Looking at your diagram, which congruence test proves that $\triangle DAX \equiv \triangle DCX$?

f Which side is equal to *AX*? Mark both sides with 3 dashes.

g Which angle is equal to $\angle DXA$?

h What is the size of $\angle DXA$? Mark this on your diagram.

i What does this prove about the diagonals of a kite? PS R C

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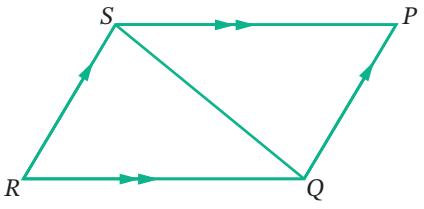
2 *SPQR* is a parallelogram, so its opposite sides are parallel.



a Why is $\angle PSQ = \angle RQS$? What type of angles are they? Copy the diagram and mark both angles with a dot.

b Which angle is equal to $\angle PQS$? Mark both angles with an arc.

c The diagonal *SQ* divides the parallelogram into 2 congruent triangles. Which test proves that they are congruent?

d Which side is equal to *SP*? Mark both sides with a dash.

e Which side is equal to *PQ*? Mark both sides with 2 dashes.

f What does this prove about the opposite sides of a parallelogram?

g Which angle is equal to $\angle P$? Mark both angles with a cross.

h What does this prove about the opposite angles of a parallelogram? PS R C

3 Draw the other diagonal *PR* of the parallelogram from question **2**, crossing *SQ* at *X*.

a Which angle is equal to $\angle SPX$? Mark both angles with little circles.

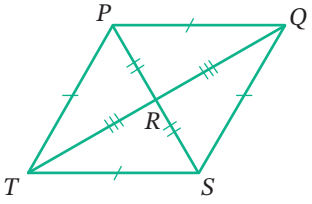
b Which test proves that the top and bottom triangles joined at *X* are congruent?

c Which side is equal to *SX*? Mark both sides with 3 dashes.

d Which side is equal to *PX*? Mark both sides with 4 dashes.

e What does this prove about the diagonals of a parallelogram? PS R C

4 *PQST* is a rhombus, so all sides are equal. A rhombus is a special type of parallelogram so its diagonals should also bisect each other as shown.



a The 2 diagonals divide the rhombus into 4 congruent triangles. Which test proves that the 4 triangles are congruent?

b What is the size of the 4 angles around point *R*? Why? Copy the diagram and mark those angles with the correct symbol.

c What does this prove about the diagonals of a rhombus?

d The bigger triangle, $\triangle PQT$, is isosceles so mark its 2 equal angles with a dot.

e Mark the other 2 angles that are equal to the angles marked with a dot.

f Mark the remaining angles in the congruent triangles with a cross since they are equal.

g What does this prove about the diagonals of a rhombus? PS R C

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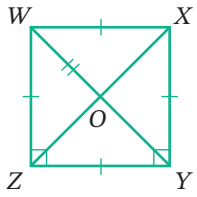
5 $WXYZ$ is a square, so all sides are equal and all angles are 90°.



- **a** Which congruence test proves that $\triangle WYZ \equiv \triangle XZY$?
- **b** Which side is equal to WY?
- **c** What does this prove about the diagonals of a square?
- **d** A square is a special type of parallelogram, so its diagonals bisect each other. Copy the diagram and mark all sides that are equal to WO with double dashes.
- **e** The 2 diagonals divide the square into 4 congruent triangles. Which test proves that they are congruent?
- **f** What is the size of the 4 angles around point O? Mark those angles with the correct symbol.
- **g** What does this prove about the diagonals of a square?
- **h** As the 4 congruent triangles are also isosceles triangles, mark all equal angles with a dot. What is the size of these angles?
- **i** What does this prove about the diagonals of a square? PS R C

Extension: Bisecting intervals and angles 8.06

EXTENSION

WS Bisecting intervals and angles

WS A page of intervals

An **interval** is a section of a line that has a definite length.

We can apply the properties of geometric figures and use geometrical instruments to bisect intervals and angles.

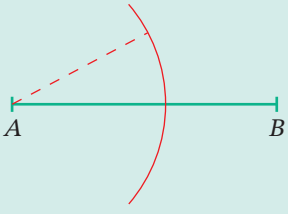
Example 8

Bisect the interval AB.

Solution

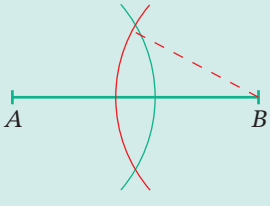
The following method involves constructing an 'invisible rhombus' whose diagonal is the interval AB. We use the property that the diagonals of a rhombus bisect each other.

Step 1:



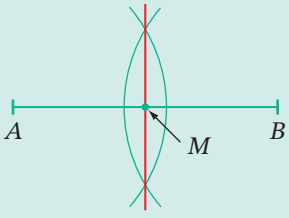
Open your compasses out to more than half the length of AB and draw a large arc from A.

Step 2:



Draw another arc the same distance from B.

Step 3:



Use a ruler and mark the midpoint M. Check with a ruler that M is halfway.

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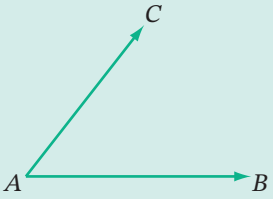
EXTENSION

Use a ruler to check that the interval has been bisected.

Example 9

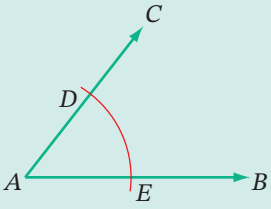
Bisect $\angle CAB$.



Solution

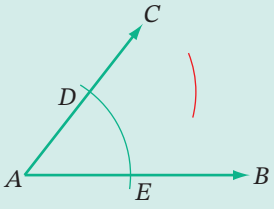
The following method involves constructing an 'invisible rhombus' with the arms of the angle being 2 sides of the rhombus, and a diagonal bisecting $\angle A$.

Step 1:



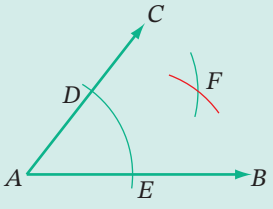
Use compasses to draw a large arc from A.

Step 2:



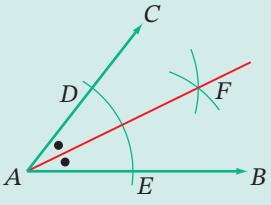
Draw another arc the same distance from D.

Step 3:



Draw another arc the same distance from E.

Step 4:



Use a ruler to join F to A to cut the angle in half.

Use a protractor to check that the angle has been bisected.

EXERCISE 8.06 ANSWERS ON P. 571

Extension: Bisecting intervals and angles UFRC

EXAMPLE 8

EXAMPLE 9

1 Draw 3 different intervals and bisect them.

2 Draw 2 acute and 2 obtuse angles and bisect them.

3 Draw a reflex angle and bisect it.

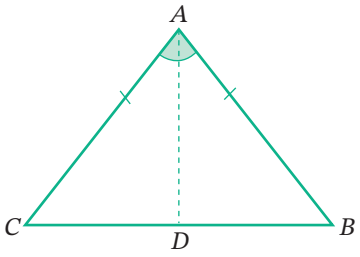
4 **a** Construct an angle of 90° by bisecting a straight angle.

b Bisect the 90° angle to form a 45° angle.

5 Draw an interval of length 10 cm and use compasses to divide it into 4 equal subintervals. Check by measuring with a ruler.

EXTENSION

6 **a** Construct an isosceles triangle and label it ABC as shown in the diagram.



b Bisect $\angle A$ and extend the line to cut BC at D.

c Which congruency test proves that $\triangle ACD \equiv \triangle ABD$?

d Hence, which angle is equal to $\angle C$? Check by measuring with a protractor.

e Measure the size of $\angle ADB$. What do you notice?

f Measure the lengths of CD and DB. What do you notice? R C

7 **a** Draw $\angle PQR$ of any size and use a pair of compasses to bisect $\angle Q$.

b Check with a protractor that $\angle Q$ has been bisected.

8.06

Mental skills 8B: Maths without calculators ANSWERS ON P. 571

Time differences

1 Study each example.

a What is the time difference between 11:40 a.m. and 6:15 p.m.?

From 11:40 a.m. to 5:40 p.m. = 6 hours

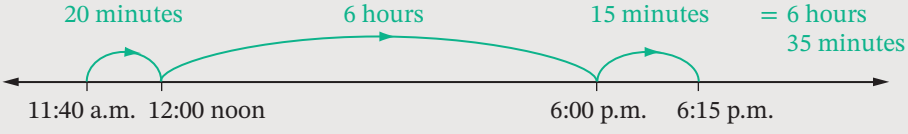
Count: '11:40, 12:40, 1:40, 2:40, 3:40, 4:40, 5:40'

From 5:40 a.m. to 6:00 p.m. = 20 min

From 6:00 p.m. to 6:15 p.m. = 15 min

5 hours + 20 min + 15 min = 6 hours 35 min

OR:



b What is the time difference between 16:45 and 23:20?

From 16:45 to 22:45 = 6 hours (22 − 16 = 6)

From 22:45 to 23:00 = 15 min

From 23:00 to 23:20 = 20 min

6 hours + 15 minutes + 20 minutes = 6 hours 35 minutes

OR:

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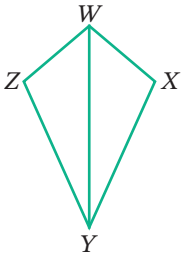
2 Now find the time difference between:

a 11:10 a.m. and 7:40 p.m.
b 6:20 p.m. and 12:00 midnight
c 4:45 p.m. and 8:10 p.m.
d 2:35 a.m. and 10:50 a.m.
e 1:05 p.m. and 12:30 a.m.
f 9:35 a.m. and 11:15 a.m.
g 04:25 and 09:35
h 14:40 and 20:25
i 7:55 a.m. and 3:50 p.m.
j 2:40 p.m. and 10:20 p.m.

Power plus ANSWERS ON P. 571

WS Congruent triangle proofs

1 Copy and complete each formal proof of congruent triangles.



a $WXYZ$ is a kite.

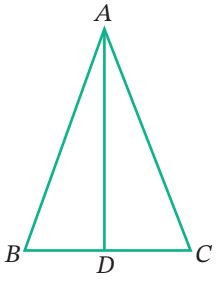
In $\triangle WYZ$ and $\triangle WYX$:

$WZ =$ _____ (equal sides of a kite)

$ZY =$ _____ (_____)

Side _____ is common to both triangles.

$\triangle WYZ \equiv \triangle WYX$ (SSS)



b ABC is an isosceles triangle, where $AB = AC$.

D is the midpoint of BC.

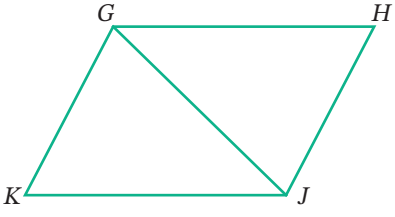
In $\triangle ABD$ and $\triangle ACD$:

$AB =$ _____ (_____)

$BD =$ _____ (D is _____)

AD is _____ to both triangles.

$\triangle ABD \equiv \triangle ACD$ (_____)



c $GHJK$ is a parallelogram.

In $\triangle GHJ$ and $\triangle JKG$:

$GH =$ _____ (_____)

$HJ =$ _____ (_____)

$\angle GHJ = \angle$_____ (_____)

$\triangle$_____ $\equiv \triangle$ _____ (_____)

2 **a** What additional property makes a parallelogram a rectangle?

b What makes a kite a rhombus?

c What makes a rectangle a square?

3 Name all quadrilaterals whose:

a opposite angles are equal
b diagonals intersect at 90°
c diagonals are equal
d angles are all 90°
e opposite sides are parallel
f diagonals bisect each other.

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CHAPTER 8 REVIEW

Language of maths

AAS	bisect	compasses	congruence test
congruent (≡)	construct	diagonal	hypotenuse
image	included angle	matching	original
reflection	RHS	rotation	SAS
SSS	superimpose	transformation	translation
vertex/vertices			

1 What are the 3 **congruence transformations**?

2 What word means:

- **a** to cut in half?
- **b** to draw accurately?
- **c** to place one shape on top of another shape so that it fits?

3 What does the phrase 'vertices must be named in matching order' mean?

4 What does **RHS** stand for:

- **a** in congruent triangles?
- **b** in solving equations?

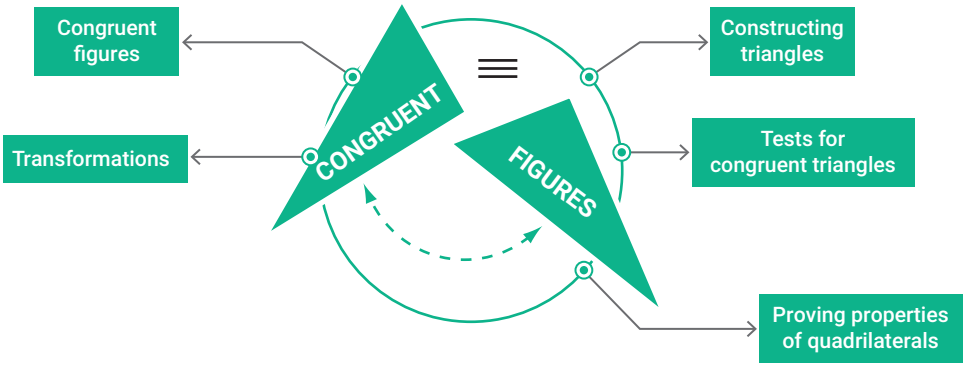
5 What is an **included angle**?

Topic summary

- How useful do you think this chapter is to you?
- Which sections did you find the most interesting? Which did you find the most difficult?

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

Mind map: Congruent figures

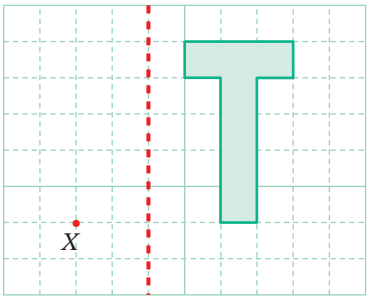


TEST YOURSELF 8

ANSWERS ON P. 571

8.01

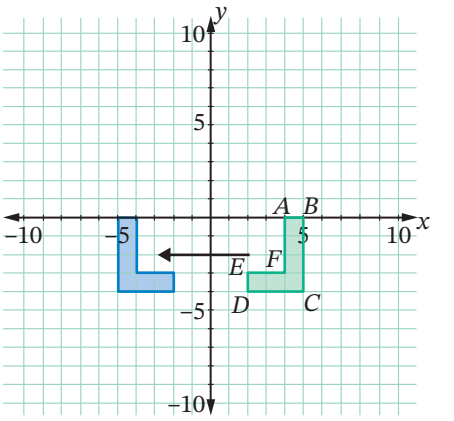
1 Copy this diagram, reflect the shape across the line, then rotate it 270° clockwise about the point X.



8.01

2 **a** Accurately describe the transformation that has been performed on the green L-shape $ABCDEF$.

b Write the coordinates of the new position of C and compare them to its original coordinates.



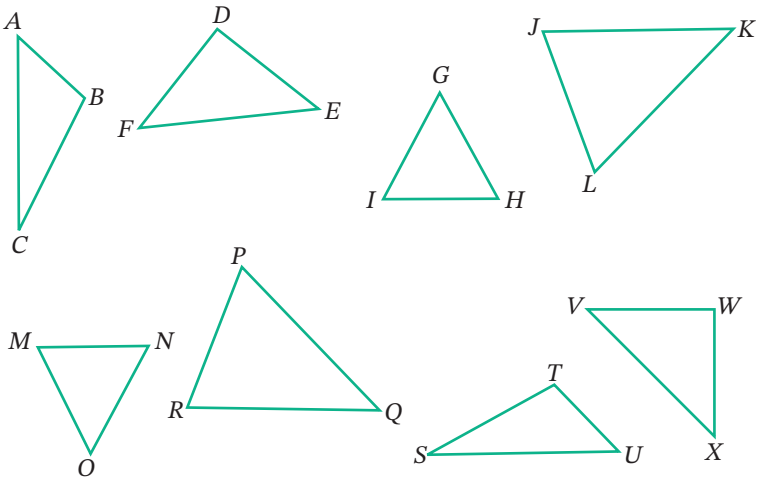
8.02

3 From the diagrams, match 4 pairs of congruent triangles and for each pair:

a name one pair of matching sides

b name one pair of matching angles

c write the congruence statement using the ≡ notation and match the order of the vertices.



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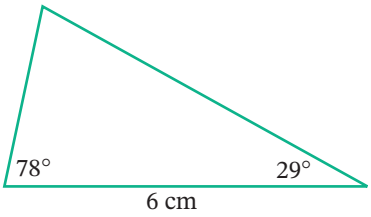
4 This triangle has a base of length 6 cm that is bounded by angles of size 78° and 29°.



8.03

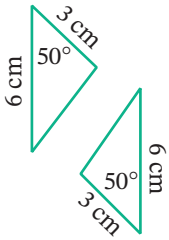
a Construct a triangle that is congruent to this triangle.

b Is it possible to construct another triangle with the same measurements that is not congruent?

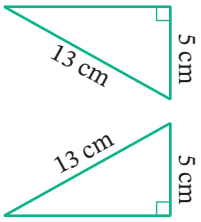
5 Which congruence test (SSS, SAS, AAS or RHS) proves that each pair of triangles is congruent?

8.04

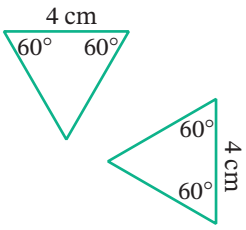
a



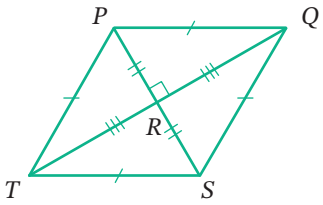
b



c



6 The diagonals of this rhombus divide the rhombus into 4 congruent triangles.



8.05

a Which test proves that the 4 triangles are congruent?

b Hence explain why the diagonals cross at right angles.

c Name the 3 angles that are equal to $\angle PQR$.

d True or false? The diagonals of a rhombus:

i are equal

ii bisect each other

iii bisect the angles of the rhombus